Estimate of Wolfenstein's Parameters ρ and η Based on a Geometry viewpoint to the Weak CP Phase

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Abstract

Based on a geometric postulation on the weak CP phase in Cabibbo-Kobayashi-Maskawa (CKM) matrix, a positive ρ is asserted. Besides, $0.18 < \eta < 0.54$ and $0.048 < \rho < 0.140$ are permitted by the present data. The corresponding geometric constraint on Wolfenstein's parameters is also worked out. We find that, according to the geometry viewpoint, η and ρ satisfy an approximate linear relation. These results can be put to the more precisely tests in near future.

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Quark mixing and CP violation is one of the most interesting and important problem in weak interaction [1-4]. In the Standard Model, They are described by the unitary Cabibbo-Kabayashi-Maskawa (CKM) matrix, which takes the following form [5-7]

$$V_{KM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{1}$$

in the standard parametrization. Here, the standard notations $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for the "generation" labels $i, j = 1, 2, 3$ being used. The real angles θ_{12}, θ_{23} and θ_{13} can all be made to lie in the first quadrant while the phase δ_{13} lies in the range $0 < \delta_{13} < 2\pi$. In following, we will fix the three angles θ_{12}, θ_{23} and θ_{13} in the first quadrant.

To make it be convenient to use the CKM matrix in the concrete calculations, Wolfenstein parametrized it as [8]

$$V_W = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{1}{2}\lambda^2) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1 + i\eta\lambda^2) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{2}$$

Actually, one can take different parametrization [7][9-13] in different cases. They are only for the convenience in discussing the different questions, but the physics does not change when adopting various parametrizations.

According to Buras etc., there is a very nice corresponding relation between Wolfenstein's parameters and the ones in the standard parametrization. It reads [14]

$$s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta). \tag{3}$$

So,

$$s_{13} = A\lambda^3\sqrt{\rho^2 + \eta^2}, \qquad \sin\delta_{13} = \frac{\eta}{\sqrt{\rho^2 + \eta^2}} \tag{4}$$

and consequently,

$$\rho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta_{13}, \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta_{13}. \tag{5}$$

In Eq.(2), λ and A are the two better known parameters. But, due to the uncertainty of hadronic matrix elements and other reasons, it is difficult to extract more information about ρ and η from experimental results. Up to now, we still know little about them. More than ten years ago, Wolfenstein estimated that the upper limit on η is about 0.1 [8], but the recent analysis indicate that, ρ and η are about [9][15]

$$-0.15 < \rho < 0.35, \qquad 0.20 < \eta < 0.45. \tag{6}$$

The central purpose of this work is to give limits on the ranges of ρ and η according to our geometric postulation on the weak CP phase.

In Ref. [16], we have found that, the weak CP phase and the other three mixing angles in CKM matrix satisfy the following geometry relation

$$\sin\delta_{13} = \frac{(1+s_{12}+s_{23}+s_{13})\sqrt{1-s_{12}^2-s_{23}^2-s_{13}^2+2s_{12}s_{23}s_{13}}}{(1+s_{12})(1+s_{23})(1+s_{13})}. \tag{7}$$

Here, we have taken δ_{13} as certain geometry phase. In fact, Eq.(7) means that, δ_{13} is the solid angle enclosed by $(\pi/2-\theta_{12})$, $(\pi/2-\theta_{13})$ and $(\pi/2-\theta_{23})$, or the area to which the solid angle corresponds on a unit sphere. Obviously, to make $(\pi/2-\theta_{12})$, $(\pi/2-\theta_{13})$ and $(\pi/2-\theta_{23})$ enclose a solid angle, the condition

$$(\pi/2-\theta_{ij})+(\pi/2-\theta_{jk}) > (\pi/2-\theta_{ki}) \quad (i\neq j\neq k\neq i.\ \ i,j,k=1,2,3.\ \ \theta_{ij}=\theta_{ji}) \tag{8}$$

must be satisfied.

Now that the four angles in CKM matrix are not independent, the four Wolfenstein's parameters A, λ, ρ and η must also be not independent.

Substituting Eqs.(3-4) into Eq.(7), it is easy to achieve

$$\frac{\eta}{\sqrt{\rho^2+\eta^2}} = 1-\frac{\lambda^2}{2}-A\lambda^3+\lambda^4(-\frac{1}{8}+A-\frac{A^2}{2}-A\sqrt{\rho^2+\eta^2}). \tag{9}$$

This is just the geometry constraint on Wolfenstein's parameters when approximate to the fourth order of λ.

It should be noted that, ρ presents as its square in the equation, hence, its sign is ambiguous. In fact, this kind of vagueness is inavoidable. Because the observables are the combinations of ρ and η rather than themselves, in which ρ always presents as its square, so, it is very difficult to specify the sign of ρ from the experimental data.

Here, we would like to give a comment on the sign of ρ. According to our geometric postulation, as discussed in Ref. [16], when fixing all the three mixing angles in the first quadrant, δ_{13} must be in the first quadrant, or at most, the fourth quadrant is permitted. In either case, Eq.(5) implies a positive ρ.

In following, let us investigate carefully the permitted ranges of ρ and η by present data. If we start out from Eq.(9) directly, and take [7][17]

$$\lambda = 0.2196\pm0.0023 \qquad A = 0.819\pm0.035$$

as inputs, then we can obtain the dependence of η on ρ. The result is shown in Fig.(1). It can be seen from the figure that, η and ρ satisfy an approximate linear relation.

We can also begin with Eq.(5). But, we should know the three mixing angles firstly. This can be arrived by use of three of the CKM matrix elements such as V_{ud}, V_{ub} and V_{tb}. In Ref. [16], we have found that, the whole matrix can be reconstructed very well based only on three of the elements and Eq.(7). Once the three mixing angles are determined, we can

extract the dependence of η on ρ again from Eq.(5). We take the relevant inputs from the data book [7]

$$V_{ud} = 0.9745 \sim 0.9760, \qquad V_{ub} = 0.0018 \sim 0.0045, \qquad V_{tb} = 0.9991 \sim 0.9993.$$

The numerical result is also shown in Fig.(1). We find it is just a little part of that drawn from Eq.(9).

Now, we can read from the figure that, when all the three inputs V_{ud}, V_{ub} and V_{tb} are taken at 95% $C.L.$, we obtain the outputs

$$0.048 < \rho < 0.140, \qquad 0.18 < \eta < 0.54. \tag{10}$$

Comparing with Eq.(6), the range for ρ has been narrowed down while the limit on η is relaxed. However, with more precise measurement on the relevant CKM matrix elements in future, we can determine them more accurately.

In conclusion, the geometry restriction for Wolfenstein's parameters is worked out. And, a positive ρ is asserted by the geometry postulation. Besides, we find $0.18 < \eta < 0.54$ and $0.048 < \rho < 0.140$ are permitted by the present data. These results can be put to the more precisely tests in near future.

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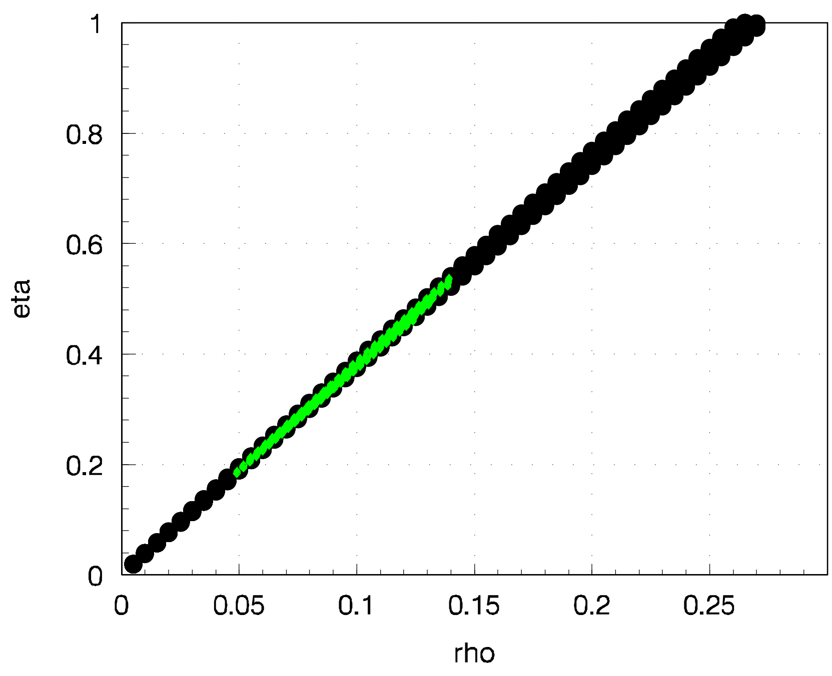
Figure 1: The dependence of η on ρ based on Eq.(9) and the permitted ranges for them by the present data. Here, the errors of the inputs have been considered.



eta
rho
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25